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EMPIRICAL FORMULA FOR THE VOLUME OF ATMOSPHERIC AIR.

BY G. W. HILL.

THE formula of Mariotte and Gay-Lussac is generally employed, in the laboratory, to reduce volumes, observed under one tension and temperature, to those which would have place under other tensions and temperatures. But Regnault, about 1845, made several series of experiments, which, if they may be relied upon, establish marked deviations from this formula. These experiments are detailed in the *Mémoires de l'Académie des Sciences de Paris*, Tom. XXI. I propose to investigate a modification of the formula, the introduction of which makes it possible to satisfy nearly these experiments.

Let T denote the temperature, here always expressed in degrees of the centigrade scale; P the tension or pressure, measured by the altitude, in metres, of a column of mercury, it is capable of supporting; the mercury being at the temperature 0 and under the action of gravity which obtains at Regnault's laboratory; and let V denote the volume. Then, for any given mass of air, these three quantities are so connected that if any two of them are assigned the remaining third is immediately determined. That is, we must have

$$\text{function}\ (V, P, T) = 0,$$

or, solved with respect to V,

$$V = \text{function}\ (P, T).$$

But the mode, in which T is to be measured, is arbitrary, and we may take atmospheric air as the thermometric substance, and assume that T increases, in direct proportion, as the volume, under constant pressure, increases. This gives

$$V = F(P) + f(P).T.$$

It is here taken for granted that, whatever may be the density of the air inclosed in the thermometer, its indications will be the same. It is true that the usual custom of experimenters has been to measure temperatures by the augmentation of tensions under constant volume; but, when Mariotte's law holds, this gives results identical with those obtained by the former method. In this case we shall have to write the equation

$$P = F(V) + f(V).T.$$

But the first equation seems preferable.

Now since, for any given constant temperature, the volume ought to be a function of the tension similar to what it is at any other temperature, it follows that, if $F(P)$ is supposed to consist of a series of terms, each of the form KP^k, where K and k are constants, so that we may write

$$F(P) = \Sigma.KP^k,$$

then we ought to have

$$f(P) = \Sigma.K_1P^k,$$

where K_1 denotes a constant, in general, different from K. Thus we should have

$$V = \Sigma.[K+K_1T]P^k.$$

The formula of Mariotte and Gay-Lussac assumes that $F(P)$ and $f(P)$ contain each only one term, in which $k = -1$. But Regnault's experiments having shown the insufficiency of this, it is in order to see whether agreement between theory and observation cannot be brought about by annexing to V an additional term, for which k has a value different from -1. Thus let us suppose that

$$V = [K + K_1T]P^{-1} + [K' + K_1'T]P^{\beta-1}$$
$$= \frac{K + K'P^\beta}{P} + \frac{K_1 + K_1'P^\beta}{P}T.$$

As V contains a factor, which is directly proportional to the mass of air considered and inversely as the unit assumed for the measurement of volumes, we prefer to write the preceding equations thus

$$V = K\left[\frac{1+\alpha'T}{P} + \frac{\alpha+\alpha''T}{P}P^\beta\right]$$
$$= K\left[\frac{1+\alpha P^\beta}{P} + \frac{\alpha'+\alpha''P^\beta}{P}T\right].$$

When the temperature is constant, the volumes are represented by the formula

$$V = K\frac{1+\alpha P^\beta}{P},$$

that is, the result from Mariotte's law must be multiplied by the factor $1+\alpha P^{\beta}$, which differs but little from unity. α is a small constant which measures the amplitude of the deviations from this law; while β is a constant exponent, which is so chosen that the more or less rapid variation of the deviations, in passing from one tension to another, may be represented as well as possible. It is evident that, in this manner, we get the utmost advantage that can be derived from the addition of a single term to V.

The experiments of Regnault may be divided into two classes; first, those where, the temperature remaining nearly constant, the volumes of the same mass of air, under different pressures, were observed; second, those where, the volumes remaining nearly the same, the tensions were observed at the temperature of freezing and boiling water. It is obvious that experiments of these two kinds extended over a sufficient range of tension would afford the data requisite for obtaining the values of the four constants α, α', α'' and β which enter into our adopted formula.

The experiments of the first class are enumerated at pp. 374—379 of the volume quoted above. As the temperature is nearly the same for all, we assume that they have been made at the average of all the noted temperatures which is 4 .747.

To save labor we may take the average of the observed volumes and tensions when they are nearly alike. In this way Regnault's 66 experiments are reduced to the 23 given in the following table. It may be noted that V is here expressed by the number of grammes of mercury required to fill the volume. The column containing $\log(PV)$ exhibits the deviation from Mariotte's law; did this law exactly hold, the numbers in this column would be identical for each series. It will be noted that, in general, they diminish with increasing pressures. The volumes being supposed to be represented by the equation

$$V = K\frac{1+\alpha P^{\beta}}{P},$$

a preliminary investigation has given the approximate values

$$\alpha = -0.0024337, \qquad \beta = 0.645.$$

With these have been computed the values of the expressions which stand at the head of the two last columns of the table, and which serve to obtain the coefficients of the equations of condition to be given presently.

As the mass of air operated on was different in each series of experiments, K will have 9 different values; it can, however, be eliminated. Taking the common logarithms of each member of the equation last given,

$$\log K + \log(1 + \alpha P) =^{\beta} \log(PV).$$

	V.	P.	No. Obs.	log (P V).	$\frac{P^\beta}{1+aP^\beta}$.	$\frac{P^\beta}{1+aP^\beta}\log P$.
First Series	1939.76	0.^{m}73899	4	3.156387	0.8244	—0.1083
	969.65	1. 47630	4	3.155790	1.2897	+0.2182
Sec'd Series	1939.37	2. 11228	3	3.612412	1.6262	0.5281
	970.40	4. 21020	3	3.611254	2.5430	1.5876
	642.82	6. 35034	2	3.610886	3.3213	2.6664
Third Series	1939.72	2. 06887	3	3.603472	1.6045	0.5066
	969.78	4. 12663	6	3.602268	2.5102	1.5452
F'rth Series	1940.65	4. 14235	2	3.905194	2.5164	1.5532
	979.78	8. 17850	3	3.903803	3.9155	3.5737
Fifth Series	1939.85	4. 21910	4	3.912988	2.5465	1.5921
	970.29	8. 40648	4	3.911516	3.9863	3.6857
	626.91	12. 98195	1	3.910545	5.2926	5.8925
Sixth Series	1940.23	6. 77001	3	4.118444	3.4623	2.8758
	970.32	13. 47353	4	4.116396	5.4226	6.1247
	685.11	19. 00213	1	4.114562	6.7913	8.6846
	675.15	19. 30191	2	4.115000	6.8612	8.8206
S'v'th Series	1941.23	6. 39003	2	4.093580	3.3347	2.6861
	969.98	12. 72859	2	4.091543	5.2248	5.7721
	633.82	19. 39954	1	4.089757	6.8842	8.8654
Eig'h Series	1940.44	9. 33401	3	4.257968	4.2676	4.1398
	970.53	18. 54702	5	4.255283	6.6842	8.4774
Ninth Series	1945.06	11. 47357	2	4.348632	4.8824	5.1740
	1053.78	21. 05700	2	4.346146	7.2643	9.6137

To reduce the matter within the treatment of the method of least squares, it will be necessary to make some assumption regarding the probable errors of the observed P and V. We will, for convenience, suppose that they are such that the function $\log(PV)$ has a probable error equal for all the observations; an assumption somewhat precarious, it is true, but it seems that we cannot easily do better.

Let the small corrections, which it is necessary to apply to the approximate values of $\log K$, a and β, be denoted by $\delta\log K$, δa and $\delta\beta$, and let us put $\delta\log K = x$, $M\delta a = y$, $a\delta\beta = z$,
where M denotes the modulus of common logarithms. $\delta\log(PV)$ being the excess of observed over calculated $\log(PV)$, we shall have the equation of condition

$$x + \frac{P^\beta}{1+aP^\beta}y + \frac{P^\beta}{1+aP^\beta}\log P.z = \delta\log(PV).$$

A little consideration will show that x will be eliminated by taking the difference of every two equations of condition arising from the same series, and attributing the weight $ww' \div \Sigma . w$ to the resulting equation, w and w' denoting the weights of the equations whose difference is taken, and $\Sigma . w$ the sum of the weights of all the equations in the series. Since the coefficients of y and z, in the equations, are all positive and nearly proportional, it will be advantageous to adopt a new unknown u, such that

$$y = u - \tfrac{5}{3}z.$$

Then the equations, with the weights that ought to be attributed them, are

1st Series	$0.4653u$	$-0.4490z$	$=$	-0.000106	Weight, 2
2nd "	0.9168	-0.4685	$=$	-0.000193	$\frac{9}{8}$
	1.6951	-0.6869	$=$	$+0.000255$	$\frac{3}{4}$
	0.7783	-0.2184	$=$	$+0.000448$	$\frac{3}{4}$
3rd "	0.9057	-0.4709	$=$	-0.000252	2
4th "	1.3991	-0.3113	$=$	$+0.000076$	1.2
5th "	1.4398	-0.3061	$=$	$+0.000038$	$\frac{16}{9}$
	2.7461	-0.2764	$=$	$+0.000432$	$\frac{4}{9}$
	1.3063	$+0.0296$	$=$	$+0.000394$	$\frac{4}{9}$
6th "	1.9603	-0.0183	$=$	$+0.000002$	1.2
	3.3290	$+0.2605$	$=$	-0.000407	0.3
	3.3989	$+0.2800$	$=$	$+0.000104$	0.6
	1.3687	$+0.2787$	$=$	-0.000409	0.4
	1.4386	$+0.2982$	$=$	$+0.000102$	0.8
	0.0699	$+0.0195$	$=$	$+0.000511$	0.2
7th "	1.8901	-0.0642	$=$	-0.000059	0.8
	3.5495	$+0.2635$	$=$	-0.000117	0.4
	1.6594	$+0.3276$	$=$	-0.000058	0.4
8th "	2.4166	$+0.3099$	$=$	-0.000164	$\frac{15}{8}$
9th "	2.3819	$+0.4699$	$=$	-0.000005	1

The derived normal equations are

$$58.672u - 0.0790z = -0.0005252,$$
$$-0.079u + 2.4453z = -0.0000157.$$

Whence

$$u = -0.000008962, \quad z = -0.000006707, \quad y = +0.000002216.$$
$$\delta\alpha = +0.0000051, \qquad \delta\beta = +0.00276.$$

Applying these corrections to the approximate values of α and β, we get the final values

$$\alpha = -0.0024286, \qquad \beta = +0.64776.$$

How well the experiments are represented by the formula, with these values of the constants, will best be seen from the following comparison of the values of $V_0 P_0 \div V_1 P_1$ given by Regnault and those computed from the formula.

Obs.	Cal.	Diff.	Obs.	Cal.	Diff.
1.001414	1.001133	+281	1.005437	1.006694	—1257
1.001448	1.001132	+316	1.005703	1.006694	—991
1.001224	1.001133	+91	1.004286	1.004777	—491
1.001421	1.001133	+288	1.004512	1.004770	—258
1.002765	1.002233	+532	1.004599	1.004779	—180
1.002759	1.002234	+525	1.004580	1.004771	—191
1.002503	1.002236	+267	1.008536	1.008106	+430
1.003539	1.004134	—595	1.008813	1.008108	+705
1.003452	1.004133	—681	1.008016	1.008286	—270
1.003309	1.004133	—824	1.008064	1.008269	—205
1.002709	1.002209	+500	1.007980	1.008288	—308
1.002724	1.002207	+517	1.004611	1.004601	+10
1.002713	1.002206	+507	1.004752	1.004601	+151
1.002528	1.002211	+317	1.008930	1.008648	+282
1.002898	1.002203	+695	1.008755	1.008642	+113
1.002762	1.002203	+559	1.006366	1.005876	+490
1.003253	1.003417	—164	1.006132	1.005880	+252
1.003090	1.003411	—321	1.006010	1.005869	+141
1.003302	1.003407	—105	1.006346	1.005878	+468
1.003336	1.003506	—170	1.005619	1.005738	—121
1.003495	1.003508	—13	1.005622	1.005736	—114
1.003335	1.003508	—173	1.005902	1.005832	+70
1.003448	1.003509	—61			

It will be seen that the differences, in the extreme cases, amount to a fourth part of the observed deviation from the law of Mariotte. Morover it is plain that some cause, which varied from series to series, has operated to vitiate these experiments, since it is possible to determine α and β so that any two series are well represented, but not possible when all the series are included in the investigation. It may be noted also that the experiments in which the original volume was reduced to one third are not in general concordant with those where the reduction was to one half.

That these discrepancies are unavoidable will be evident from the following exposition. Let us put

$$\text{com. log}\,(PV) = F(P).$$

The observations of Regnault may be resume'ed in the following 9 results, all formed by combining tolerably concordant data,

1 . . . . $F(1.476)—F(0.739) = 0.000598,$

2 . . . . $F(4.168)—F(2.091) = 0.001181,$

3 . . . . $F(6.350)—F(2.112) = 0.001526,$

4 . . . . $F(8.292)—F(4.182) = 0.001437,$

5 . . . . $F(12.982)-F(4.219) = 0.002443$,
6 . . . . $F(13.101)-F(6.580) = 0.002042$,
7 . . . . $F(19.276)-F(6.580) = 0.003743$,
8 . . . . $F(18.547)-F(9.334) = 0.002685$,
9 . . . . $F(21.057)-F(11.474) = 0.002486$.

These are the data actually furnished by Regnault for the determination of the function $F(P)$. Employing the graphical method, we endeavor to construct the curve whose equation is $y = F(x)$. One of the special values of $F(x)$, as $F(0.739)$, may be taken arbitrarily, and then the value of $F(1.476)$ becomes known. This premised, we see that each of the 9 equations furnishes the length, direction and abscissæ of the extremities of a chord, of the sought curve. Placing the chord, corresponding to the first equation, arbitrarily, and drawing the others on any part of the paper, but with the correct direction, and correct abscissæ of their extremities, we endeavor, by imparting a motion to all their points parallel to the axis of y, to make them fall into line as the chords of the same continuous curve. We find that if we take 1, 2, 4, 6 and 7, they can be made to indicate a tolerably continuous curve; but then 3, 5, 8 and 9 are not satisfied.

Again, from this graphical process, we see that there cannot be much variation of curvature between the extremities of each chord, and hence the tangent to the curve, corresponding to the abscissa, which is the mean of the abscissæ of the extremities, ought to be, very approximately, parallel to the chord; or in other terms

$$\frac{d}{dx}F\left(\frac{x_1+x_0}{2}\right) = \frac{F(x_1)-F(x_0)}{x_1-x_0}.$$

This gives the following values of $\frac{dy}{dx}$,

	x.	$\frac{dy}{dx}$.
1	1.108	+0.0008113
2	3.130	0.0005686
3	4.231	0.0003770
4	6.237	0.0003497
5	8.600	0.0002788
6	9.840	0.0003131
7	12.428	0.0002948
8	13.940	0.0002914
9	16.265	+0.0002594

From the general course of these values of $dy \div dx$ it may be gathered that this function, at first, diminishes rapidly, afterwards more slowly, and then tends, with higher values of x, to become nearly constant. But while this is the conclusion from the *tout ensemble*, a comparison of some of the values contradicts it. Thus from 1, 2 and 3, while $dy \div dx$ diminishes 0.0002427 in an interval 2.0 in x, it afterwards diminishes 0.0001916 in an interval 1.1 of x. All attempts then to represent these data by a curve, without singular points, must, evidently, show large errors.

For the discussion of the second class of experiments, let us assume that α has the signification we have given it in the general formula for V. Then the volume remaining the same, if P_0 and P_1 denote the tensions observed respectively at 0° and 100°, we have

$$\frac{P_1}{P_0} = \frac{1+100\alpha'+(\alpha+100\alpha'')P_1{}^{\beta}}{1+\alpha P_0{}^{\beta}}.$$

$P_1 \div P_0$ is the quantity Regnault has designated by $1+100\alpha$, let us denote it by A; then if, for convenience, we put

$$\gamma = 1+100\alpha', \qquad \gamma' = \alpha+100\alpha'',$$

each determined value of A will give the equation of condition

$$\gamma + P_1{}^{\beta}.\gamma' = A + AP_0{}^{\beta}.\alpha.$$

The following are Regnault's determinations of A augmented, in general, by 0.00018, for the reason that we adopt the mean coefficient 0.00018153 for the expansion of mercury between 0° and 100°, found by this experimenter, instead of the value $\frac{1}{5550}$ used by him, (See Note, p. 31 of the vol.); the last column contains the page of the volume, where the experiments may be found.

P_0.	P_1.	A.	No. Obs.	Obs.-Cal.	Page.
$0^{\text{m}}.110$	$0^{\text{m}}.149$	1.36500	10	—0.00012	99
0 .174	0 .237	1.36531	3	—0.00004	99
0 .266	0 .362	1.36560	2	—0.00003	99
0 .375	0 .510	1.36598	4	+0.00005	99
0 .548	0 .746	1.36673	3	+0.00038	57
0 .756	0 .7535	1.36724	4	+0.00035	66
0 .557	0 .754	1.36651	18	+0.00014	43
0 .656	0 .757	1.36641	14	—0.00022	33
0 .747	1 .016	1.36663	3	—0.00014	58
0 .771	1 .049	1.36696	11	+0.00014	51
1 .678	2 .286	1.36778	2	—0.00059	109
1 .693	2 .306	1.36818	4	—0.00021	109
2 .526	2 .517	1.36962	2	—0.00018	114
2 .622	2 .614	1.36982	2	—0.00011	114
2 .144	2 .924	1.36912	2	+0.00007	109
3 .656	4 .992	1.37109	4	+0.00031	109

Adopting, for convenience, as an unknown in the place of γ,

$$x = \gamma + \gamma' - 1.367,$$

we have the following equations, to each of which we attribute a weight equal to a tenth of the number of experiments it is founded upon:

$$\begin{aligned}
x - 0.7086\gamma' - 0.3268a &= -\ 0.00200 \\
x - 0.6064\gamma' - 0.4398a &= -\ 0.00169 \\
x - 0.4821\gamma' - 0.5790a &= -\ 0.00140 \\
x - 0.3534\gamma' - 0.7237a &= -\ 0.00102 \\
x - 0.1728\gamma' - 0.9258a &= -\ 0.00027 \\
x - 0.1674\gamma' - 1.1410a &= +\ 0.00024 \\
x - 0.1670\gamma' - 0.9354a &= -\ 0.00049 \\
x - 0.1650\gamma' - 1.040\ a &= -\ 0.00059 \\
x + 0.0105\gamma' - 1.131\ a &= -\ 0.00037 \\
x + 0.0317\gamma' - 1.155\ a &= -\ 0.00004 \\
x + 0.708\ \gamma' - 1.913\ a &= +\ 0.00078 \\
x + 0.718\ \gamma' - 1.924\ a &= +\ 0.00118 \\
x + 0.818\ \gamma' - 2.496\ a &= +\ 0.00262 \\
x + 0.863\ \gamma' - 2.558\ a &= +\ 0.00282 \\
x + 1.004\ \gamma' - 2.243\ a &= +\ 0.00212 \\
x + 1.834\ \gamma' - 3.175\ a &= +\ 0.00409
\end{aligned}$$

The derived normal equations, for determining x and γ', are

$$\begin{aligned}
x - 0.0047\gamma' - 1.162a &= -\ 0.000144, \\
-\ 0.0415x + 2.9547\gamma' - 3.373a &= +\ 0.007074,
\end{aligned}$$

whence

$$\begin{aligned}
x &= -\ 0.000133 + 1.168a, \\
\gamma' &= +\ 0.002392 + 1.158a,
\end{aligned}$$

and

$$\begin{aligned}
a' &= +\ 0.00364475 + 0.00010a, \\
a'' &= +\ 0.00002392 + 0.00158a.
\end{aligned}$$

The equation which determines a has already been obtained from the discussion of the first class of experiments; it is

$$\frac{a + 4.747a''}{1 + 4.747a'} = -\ 0.0024286.$$

The last three equations being solved, we gather that the volume of any mass of air is represented by the formula

$$V = \frac{K}{P}\left[1+aP^{\beta}+(a'+a''P^{\beta})T\right],$$

in which

$$a = -\,0.002565, \qquad a' = +\,0.0036445,$$
$$a'' = +\,0.00001987, \qquad \beta = 0.64776.$$

How well the second class of experiments is satisfied by this formula may be seen from the numbers in the column headed Obs.-Cal.

If we have $T = \frac{0.002565}{0.00001987} = 129°.1$, V takes the form

$$V = \frac{K}{P}.$$

Hence we have the noteworthy result that:—

About the temperature 130°, *air follows quite exactly the law of Mariotte.* For the following temperatures and pressures the volume vanishes:—

T.	P.
0°	$9995^{m}.49$
— 50	4420 .13
—100	2048 .00
—150	896 .26
—200	314 .23

These numbers may be regarded as indications of the magnitude of pressure necessary for the condensation of air. The table is in accordance with the well-known fact that reduction of temperature facilitates condensation.

A table is given below which will be found useful in the application of the formula. It contains the functions $\log(1+aP^{\beta})$ and $\frac{a'+a''P\beta}{1+aP\beta}$, the latter being the coefficient of expansion under a constant pressure.

As an example let us suppose that the volume of a mass of air has been observed under the pressure $2^{m}.5$ and the temperature 20°; it is required to find the factor necessary for reducing it to the pressure $0^{m}.76$ and temperature 0°. From the table we get 3.07064. By employing the ordinary formula with the coefficient 0.003665 of expansion, there is obtained 3.06-482, which differs from the preceding by about a 525th part.

Rigorously observations of pressure made in localities having an intensity of gravity different from that which prevails at Regnault's laboratory ought to be multiplied by the ratio of the former to the latter. The latitude of Regnault's laboratory is stated at 48° 50′ 14″, the altitude above sea level at 60^{m}, and the intensity of gravity at $9^{m}.8096$.

P.	$\log(1+aP^\beta)$.		Coeff. of Exp.		P.	$\log(1+aP^\beta)$.		Coeff. of Exp.	
$0^m.0$	0.000000	251	0.0036445	66	$7^m.0$	9.996053	181	0.0037485	48
0 .1	9.999749	142	36511	37	7 .5	9.995872	177	37533	47
0 .2	9.999607	118	36548	31	8 .0	9.995695	174	37580	46
0 .3	9.999489	105	36579	28	8 .5	9.995521	169	37626	45
0 .4	9.999384	96	36607	25	9 .0	9.995352	167	37671	44
0 .5	9.999288	89	36632	24	9 .5	9.995185	164	37715	43
0 .6	9.999199	84	36656	22	10 .0	9.995021	161	37758	43
0 .7	9.999115	80	36678	20	10 .5	9.994860	158	37801	42
0 .8	9.999035	77	36698	21	11 .0	9.994702	156	37843	42
0 .9	9.998958	73	36719	19	11 .5	9.994546	153	37885	40
1 .0	9.998885	71	36738	18	12 .0	9.994393	151	37925	40
1 .1	9.998814	69	36756	19	12 .5	9.994242	149	37965	40
1 .2	9.998745	67	36775	18	13 .0	9.994093	147	38005	39
1 .3	9.998678	65	36793	16	13 .5	9.993946	146	38044	39
1 .4	9.998613	64	36809	17	14 .0	9.993800	143	38083	38
1 .5	9.998549	62	36826	16	14 .5	9.993657	142	38121	38
1 .6	9.998487	61	36842	16	15 .0	9.993515	141	38159	37
1 .7	9.998426	59	36858	16	15 .5	9.993374	138	38196	37
1 .8	9.998367	59	36874	15	16 .0	9.993236	138	38233	36
1 .9	9.998308	57	36889	15	16 .5	9.993098	136	38269	37
2 .0	9.998251	272	36904	73	17 .0	9.992962	134	38306	36
2 .5	9.997979	254	36977	67	17 .5	9.992828	133	38342	35
3 .0	9.997725	240	37044	62	18 .0	9.992695	132	38377	35
3 .5	9.997485	228	37106	60	18 .5	9.992563	131	38412	35
4 .0	9.997257	218	37166	58	19 .0	9.992432	129	38447	35
4 .5	9.997039	210	37224	56	19 .5	9.992303	129	38482	34
5 .0	9.996829	203	37280	53	20 .0	9.992174	127	38516	34
5 .5	9.996626	196	37333	52	20 .5	9.992047	126	38550	34
6 .0	9.996430	191	37385	51	21 .0	9.991921	126	38584	34
6 .5	9.996239	186	37436	49	21 .5	9.991795		38618	

ON ADJUSTMENT FORMULAS.

BY E. L. DE FOREST, M. A.

(*Continued from page* 86.)

THREE separate tests of good adjustment have been proposed by the present writer, for which see *Smithsonian Report* of 1873, p. 333, and a more recent essay in pamphlet form; (*Interpolation and Adjustment of Series*, New Haven, 1876). The first of these tests is, that if we denote by v the residual error, found by subtracting an adjusted term from the correspoding